

Refinement of Forest Vegetation Simulator Individual Tree Model Growth and Yield Model for the Acadian Region

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Introduction

This CFRU project was initiated in October of 2008 and been a primary research focus since then. The project has basically involved compiling a database of permanent regional growth and yield plots and refitting the various component equations that currently comprise the Forest Vegetation Simulator (FVS) individual tree growth and yield model. Previous CFRU reports have presented the climate site index metric of site productivity, height to diameter, height to crown base, and diameter as well height increment equations. These equations continue to be evaluated and refined. Current model forms and parameter estimates are presented in this report. In addition, the current approach for predicting stand- and tree-level mortality is also presented. Finally, these equations are being incorporated into a software system that will be used to conduct model projections and summaries. This software system is presented and discussed.

Methods

The database compiled and presented in Weiskittel et al. (2010) been extensively cleaned and reformatted. Due to unresolved issues with some of the original data, certain datasets have currently been excluded. The current datasets being used for modeling are the permanent sampling plots from New Brunswick, Nova Scotia, and Quebec, the US Forest Service Forest Inventory and Analysis plots, and long-term plots at the Penobscot Experimental Forest. This results in a database of 1,751,798 and 897,384 observations of individual tree diameter at breast height (DBH) and height (HT).

Total height

The total height equation of Rijal et al. (2012b) was generalized and extended to more species. The equation is:

$$[1] \quad HT = 1.37 + \left((b_0 + d_{sp}) + CSI^{b_1} \right) * \left(1 - \exp(-b_2 * DBH) \right)^{(b_3 + e_{sp} + b_4 * \ln(CCF + 1) + b_5 * BAL)}$$

where HT is total tree height in m, CSI is climate site index (m), DBH is diameter at breast height in cm, CCF is crown competition factor of Krajicek et al. (1961) computed using the maximum

crown width equations of Russell and Weiskittel (2011), BAL is tree basal area in larger trees ($m^2 ha^{-1}$), the b_i 's are parameters estimated from the data, and d_{sp} and e_{sp} are species specific parameters also estimated from the data.

Height to crown base

Similar to the total height equation, the height to crown base equation of Rijal et al. (2012a) was generalized and extended to more species. The equation is:

$$[2] \quad HCB = \frac{HT}{\left(1 + \exp\left((b_0 + d_{sp}) + b_1 * DBH + b_2 * HT + b_3 * \left(\frac{DBH}{HT}\right) + b_4 * \ln(CCF + 1) + b_5 * BAL\right)\right)^{\left(\frac{1}{6}\right)}}$$

where HCB is height to crown base in m and all other variables have been defined above.

Diameter increment

Individual tree diameter increment was modeled as basal area increment with the following equation:

$$[3] \quad \Delta BA = (b_0 + d_{sp}) * \left(BA^{((b_1 + e_{sp}) + b_2 * CSI)} \right) * \exp\left(\left((b_3 + f_{sp}) + b_4 * BAL_{SW} + b_5 * BAL_{HW}\right) * BA\right)$$

where ΔBA is the basal area increment ($m^2 yr^{-1}$), BA is the tree initial basal area (m^2), BAL_{SW} is the basal area in larger softwood trees ($m^2 ha^{-1}$), BAL_{HW} is the basal area in larger hardwood trees ($m^2 ha^{-1}$), and f_{sp} is the species specific parameter.

Height increment

Individual tree height increment was modeled with the following equation:

$$[4] \quad \Delta HT = \exp\left(\frac{\left((b_0 + d_{sp}) + (b_1 + e_{sp}) * \ln(HT) + b_2 * HT + b_3 * BAL + b_4 * \ln(CR)\right)}{b_5 * \ln(CSI) + b_6 * \sqrt{BA}}\right)$$

where ΔHT is the annual height increment ($m yr^{-1}$), CR is crown ratio (0-1), and all other variables have been previous defined.

Mortality

For predicting mortality, a variety of approaches were initially evaluated and relatively poor performance was observed. Consequently, a three-stage approach was developed and used to predict the probability and amount of mortality at the stand-level, which is then allocated to individual trees. Each stage is described separately below.

Stage 1: Probability of Mortality

The probability that a plot experiences mortality was fitted using a general logistic equation form:

$$[5] \quad \text{Pr}(\text{Mortality}) = \frac{e^{f(X)}}{1+e^{f(X)}}$$

where $f(X)$ is a linear combination of independent variables. Boosted regression (REF) was used to identify potential independent variables and a number of equations fitted using backward and forward elimination/addition until all variables in the equation were significant and no additional variables further reduced the root mean square error (RMSE).

The final form of $f(X)$ was:

$$[6] \quad f(X) = b_0 + b_{0,R} + b_1 * BA_T + b_2 * BA_T^2 + b_3 * \Delta BA_{30} + b_4 * BA_{BF} + b_5 * BA_{IH}$$

where BA_T is the total stand basal area ($m^2 ha^{-1}$), ΔBA_{30} is the annual basal area survivor growth of the largest trees within the stand that summed to a relative density of 0.30 ($m^2 ha^{-1} yr^{-1}$), QMD is the quadratic mean diameter (cm), BA_{BF} is the basal area of balsam fir ($m^2 ha^{-1}$), BA_{IH} is the basal area of intolerant hardwoods ($m^2 ha^{-1}$), and the $b_{j,R}$ were region-specific random effects (Maine, Nova Scotia, New Brunswick, Quebec). Equation 5 was the used to predict probability of mortality for each plot at each annualized measurement step.

Stage 2: Basal Area Mortality Prediction

Using only those plots in which mortality was observed, a nonlinear mixed effects model was fitted to predict basal area mortality (BA_{MORT} , $m^2 ha^{-1} yr^{-1}$), again using region was a random effect. Boosted regression was initially used to identify variables that potentially influenced BA_{MORT} , then these variables were tested in a number of equation forms typically found in the literature for predicting mortality. The final equation form selected was:

$$[7] \quad BA_{MORT} = \left(b_0 + b_{0,R} + b_1 * (BA_{BF}/BA_T) + b_2 * (BA_{IH}/BA_T) \right) (BA_T)^{(b_3 + b_{3,R} + b_4 \Delta BA_{30})} + (b_5 + b_{5,R}) * BA_{BF}^{(b_6 + b_{6,R} + b_7 (QMD_{BF}/QMD))}$$

where QMD_{BF} is the quadratic mean diameter of balsam fir and all of the variables have been previously defined above.

Stage 3: Individual Tree Mortality Prediction

To allocate the predicted BA_{MORT} to individual trees, a logistic regression equation linked with a right censored three parameter Weibull was used. The equation is:

$$[8] \quad \text{Pr}(\text{Survival}) = \frac{e^{(b_0 + d_{sp} + (b_1 + e_{sp}) * \text{DBH} + b_2 * \text{DBH}^2 + b_3 * \text{BAL} + b_4 * (\frac{\text{DBH}}{\text{QMD}}) + b_5 * \Delta \text{BA}_{30} + b_6 * \text{BA}_T)}}{1 + e^{(b_0 + d_{sp} + (b_1 + e_{sp}) * \text{DBH} + b_2 * \text{DBH}^2 + b_3 * \text{BAL} + b_4 * (\frac{\text{DBH}}{\text{QMD}}) + b_5 * \Delta \text{BA}_{30} + b_6 * \text{BA}_T)}} * K$$

where Pr(Survival) is the probability of tree survival and K is defined as:

$$[9] \quad K = \begin{cases} 1 & \text{if } \text{DBH} < 40 \\ e^{-\left(\frac{\text{DBH} - 40}{b}\right)^c} & \text{if } \text{DBH} \geq 40 \end{cases}$$

where b and c are the scale and shape parameters of the Weibull distribution, respectively.

Results

Total height

A total of 365,380 observations of total height were available for analysis. Equation 1 explained 72.6% of the original variation in total height and the inclusion of the species-specific parameters increased this to 77.8%. All parameters were statistically significant and had a biologically logical sign (Table 1). For a given set of covariates, quaking aspen was the tallest species, while eastern hemlock was the shortest (Figure 1).

Height to crown base

A total of 269,255 observations of height to crown base were available for analysis. Equation 2 explained 65.0% of the original variation in height to crown base and the inclusion of the species-specific parameters increased this to 67.0%. All parameters were statistically significant and had a biologically logical sign. For a given set of covariates, yellow birch and black spruce had the highest height to crown base, while eastern hemlock had the lowest (Figure 1).

Diameter and height increment

A total of 504,689 and 88,956 observations of diameter and height increment were available for analysis, respectively. Equation 3 explained 45.0% of the original variation in diameter increment and the inclusion of the species-specific parameters increased this to 47.2%. Equation 4 explained 25.2% of the original variation in height increment and inclusion of the species-specific parameters increased this to 26.1%. All parameters were statistically significant and had a biologically logical sign. For given a level of tree and stand variables, white pine and black spruce showed the highest and lowest diameter increment, respectively, while species differences in height increment were not great (Figure 2).

Mortality

The final dataset had 1,171,515 tree- and 150,763 plot-level observations. Overall, individual tree mortality was relatively rare as less than 10% of the trees were classified as dead and only

33% of the plot remeasurement periods had a mortality event. Across the entire dataset, BA_{MORT} averaged $0.35 \text{ m}^2 \text{ ha}^{-1} \text{ yr}^{-1}$, while it was closer to $0.45 \text{ m}^2 \text{ ha}^{-1} \text{ yr}^{-1}$ when only plots experiencing mortality were evaluated. Despite the noisiness of the underlying data, Equations 6-8 fit the data well with all parameters being statistically significant and with a biologically logical sign.

For the stand-level probability of mortality model, the RMSE was 0.37 and had a R^2 of 16.1%. There were significant regional differences in stand-level probability of mortality trends (Figure 3). Overall, the most influential variable was ΔBA_{30} followed by BA_T . All of the influential factors had a positive relationship with stand-level probability of mortality.

The stand-level basal area mortality model had a R^2 of 23% and again highlights significant regional differences. Overall, mortality rates for a given stand structure and composition were highest in New Brunswick followed by Maine, Nova Scotia, and Quebec (Figure 4). Like the stand-level probability of mortality model, the most influential variable was ΔBA_{30} followed by BA_T . Basal area mortality increased with both greater percent balsam fir basal area and with a higher ratio of balsam fir QMD to overall stand QMD; though the effect of increasing percent balsam fir had a greater effect.

The tree-level mortality model had a R^2 of 9.2% and residual standard error of 0.29. All parameters were statistically significant and had a biological logical sign. There were significant differences between the species (Table 4). For given tree and stand conditions, white pine showed a higher probability of annual mortality, while black spruce showed the lowest (Figure 5).

Discussion

With the finalization of these equations, all of the necessary component equations have been fit and are currently being inserted into the Open Stand Model (OSM) being developed by Dr. Chris Hennigar (Figure 6). The OSM is a highly flexible software framework that will allow rapid incorporation of future alterations of the model, the ability to link to other existing software, and capability to process stands rapidly with high degree of user control. A more complete description of OSM was provided to the CFRU in September 2012 by Dr. Hennigar. Consequently, this Discussion will focus solely on the performance of the equations.

The generalization of the total height and height to crown base equations extended them to other minor species in the region and provide a more robust prediction for the more common species. Local calibration of these equations will be possible when measurements are available and is highly recommended. The diameter increment equation is currently being refitted as Russell et al. (2011) found that long-term predictions using basal area increment rather than diameter increment can increase model bias. The diameter increment will likely be similar in form to the current height increment equation. Despite showing a reasonable fit to the data, the height increment equation is not showing very distinct species differences in predictions.

Further evaluation will be necessary to ensure it is behaving correction across the range of the available data.

Mortality will be the most difficult model component to improve. The three-stage approach outlined here is fairly robust and has drastically improved predictions. However, the overall performance for a given stand will like be poor due to the complex array of factors that influence mortality with many of them being highly stochastic. The analysis also highlighted significant differences in mortality patterns between the different regions evaluated. Maine often had a mortality behavior in between the other Canadian provinces, which is likely due to differences in past management and disturbance histories. The stand-level mortality models clearly show the sensitivity of mortality to stand density. In addition, the models are also dependent on stand growth, which should help the model to be capable of representing stand dynamics during stagnation. At the tree-level, species differences in mortality patterns were evident. White pine was predicted to be the most likely to die for a given set of covariates, but the probability was only slightly higher than other species. Continued evaluation of the equation and the overall model predictions will be necessary to ensure proper behavior.

Future efforts will work on evaluating system behavior as the equations have generally been fit independently and between equations interactions can cause strange behavior. The release of the beta version of OSM will allow users to test run and provide feedback on the model behavior. Continued efforts to improve equation fits, increase representation of different site factors like soil drainage and aspect, and extend the model to managed stand conditions.

Literature Cited

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Table 1. Generalized parameter estimates and standard errors (SE) for the total height (Eqn. 1), height to crown base (Eqn. 2), diameter increment (Eqn. 3), and height increment (Eqn. 4) equations.

Parameter	Total Height		Height to Crown Base		Diameter Increment		Height Increment	
	Estimate	SE	Estimate	SE	Estimate	SE	Estimate	SE
b ₀	12.44847305	0.389041	0.29070	0.17514067	0.0195744	0.00441	0.6644887	0.45666
b ₁	0.801705832	0.001222	0.00636	0.001816315	0.6890597	0.029771	-1.715156	0.27303
b ₂	0.043617034	0.000225	-0.02288	0.002556165	-0.000013	0.000008	0.1139081	0.02068
b ₃	1.048674338	0.039192	0.08232	0.02387724	-2.099691	0.006219	-0.013541	0.00132
b ₄	0.011483716	0.000324	-0.03086	0.004257092	-0.202029	0.000403	-0.626488	0.03081
b ₅	-0.00755099	4.26E-05	-0.01701	0.000384455	-0.141399	0.000563	0.1381055	0.01574
b ₆	-	-	-	-	-	-	0.0058419	0.00001

Table 2. Species specific parameter estimates for the total height (Eqn. 1), height to crown base equations (Eqn. 2), diameter increment (Eqn. 3), and height increment (Eqn. 4).

Species	Total Height		Height to Crown Base	Diameter Increment			Height Increment	
	d _{sp}	e _{sp}	d _{sp}	d _{sp}	e _{sp}	f _{sp}	d _{sp}	e _{sp}
American beech	-0.60586	-0.08105	-0.61448	-0.03216	-0.07583	-2.505	-0.0533	0.0206
Balsam fir	0.874025	0.161628	0.17913	-0.02386	0.059918	-1.96717	0.1237	-0.1102
Black spruce	1.81723	0.176327	-0.67838	-0.03763	-0.13488	3.176286	0.1472	-0.1116
Eastern hemlock	-0.24547	0.225872	1.04882	-0.02129	0.086307	-0.67182	-0.2719	0.1645
Jack pine	6.322689	0.504639	-0.86368	-0.00963	0.315134	-10.0582	0.3575	-0.1707
Paper birch	1.437664	-0.05351	-0.52524	-0.03576	-0.08039	-1.66648	-0.1104	0.0355
Quaking aspen	2.635654	-0.02573	0.24077	-0.03114	-0.09987	1.417473	-0.4196	0.2995
Red maple	0.538154	-0.12317	-0.55332	-0.0312	0.007209	0.101817	0.3750	-0.2335
Red spruce	1.668609	0.201859	-0.40110	-0.03012	0.01523	0.389315	-0.1465	0.0949
Sugar maple	0.903342	-0.15214	-0.46608	-0.02674	0.072611	-0.60242	0.4608	-0.2566
White pine	0.910013	0.213712	-0.45910	-0.01588	0.107642	0.049527	0.0521	0.0393
White spruce	1.419979	0.290358	0.17039	-0.02269	0.072533	-2.89254	0.2094	-0.1757
Yellow birch	-0.69698	-0.18123	-0.04040	-0.03242	-0.08005	-0.78848	0.5100	-0.2797

Table 3. Generalized parameter estimates and standard errors (SE) for the probability of stand mortality (Eqn. 6), basal area mortality amount (Eqn. 7), and individual tree mortality probability (Eqn. 8) prediction equations.

Parameter	Probably of stand mortality (Eqn. 5)		Basal area mortality amount (Eqn. 6)		Individual tree mortality probability (Eqn. 7)	
	Estimate	SE	Estimate	SE	Estimate	SE
b ₀	-0.6959509	0.3462012	0.2554043	0.10023921	2.224763	0.14843420
b ₁	0.0703694	0.0024818	0.2315199	0.00481723	0.076734	0.00933890
b ₂	-0.0009841	0.0000441	0.0202025	0.00249382	-0.000907	0.00002928
b ₃	0.7818633	0.0164109	0.5067010	0.09433632	-0.024431	0.00057838
b ₄	0.0486126	0.0022536	-2.0370423	0.02150724	-0.311719	0.01054643
b ₅	0.0325371	0.0018925	0.0781800	0.02414730	3.402583	0.02181862
b ₆	-	-	0.3453608	0.05176340	0.014305	0.00049544
b ₇	-	-	0.0995085	0.01632631	-	-
b _{0,Maine}	-0.7768921	-	-0.069619	-	-	-
b _{0, New Brunswick}	-0.3874579	-	0.3433697	-	-	-
b _{0, Nova Scotia}	0.0985491	-	-0.1251712	-	-	-
b _{0, Quebec}	1.0658010	-	-0.1485785	-	-	-
b _{3,Maine}	-	-	0.06072934	-	-	-
b _{3, New Brunswick}	-	-	-0.3178151	-	-	-
b _{3, Nova Scotia}	-	-	0.08274501	-	-	-
b _{3, Quebec}	-	-	0.17434071	-	-	-
b _{5,Maine}	-	-	-0.0100276	-	-	-
b _{5, New Brunswick}	-	-	0.0678903	-	-	-
b _{5, Nova Scotia}	-	-	0.00860064	-	-	-
b _{5, Quebec}	-	-	-0.0664633	-	-	-
b _{6,Maine}	-	-	-0.010830	-	-	-
b _{6, New Brunswick}	-	-	-0.016878	-	-	-
b _{6, Nova Scotia}	-	-	-0.1210011	-	-	-
b _{6, Quebec}	-	-	0.14871012	-	-	-

Table 4. Species specific parameter estimates for the individual tree mortality equation (Eqn. 8).

Species	d _{sp}	e _{sp}
American beech	0.938413880	-4.0560e-02
Balsam fir	1.032042586	-8.5124e-02
Black spruce	1.562311307	-5.9391e-02
Eastern hemlock	1.468455064	-1.6517e-03
Jack pine	-0.113945145	3.7824e-02
Paper birch	0.783286007	-2.8016e-02
Quaking aspen	-0.756473287	1.9419e-02
Red maple	0.789828397	1.1527e-02

Red spruce	0.901504617	-2.1202e-02
Sugar maple	0.840389245	3.0382e-02
White pine	0.210710332	2.2268e-02
White spruce	0.772213691	-3.2541e-02
Yellow birch	0.962532973	-1.6120e-03

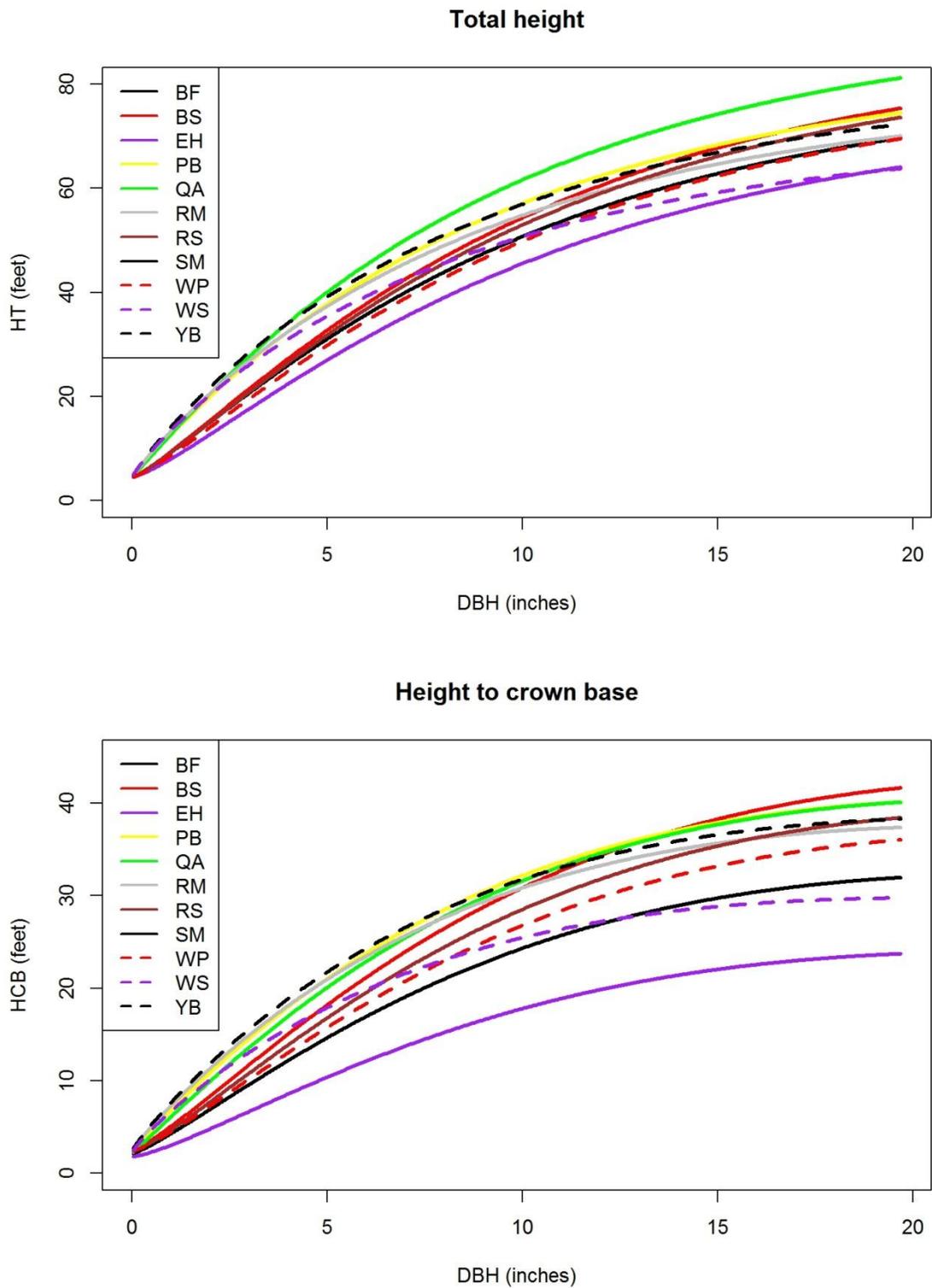


Figure 1. Predicted total height (top) and height to crown base (bottom) over diameter at breast height using Equations 1 and 2, respectively, for an open-grown tree.

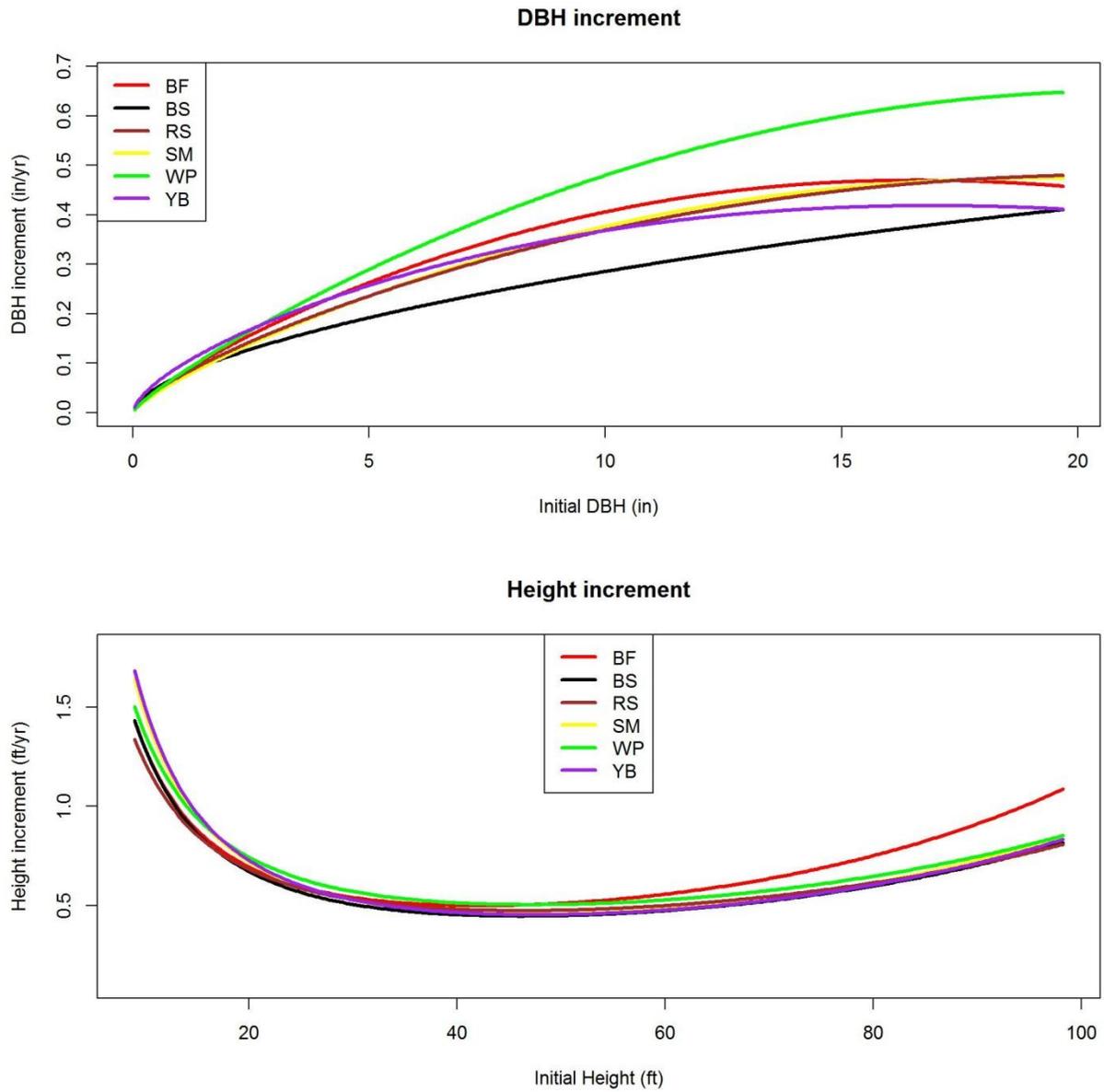


Figure 2. Predictions of diameter (top) and height increment for different species for an open-grown tree.

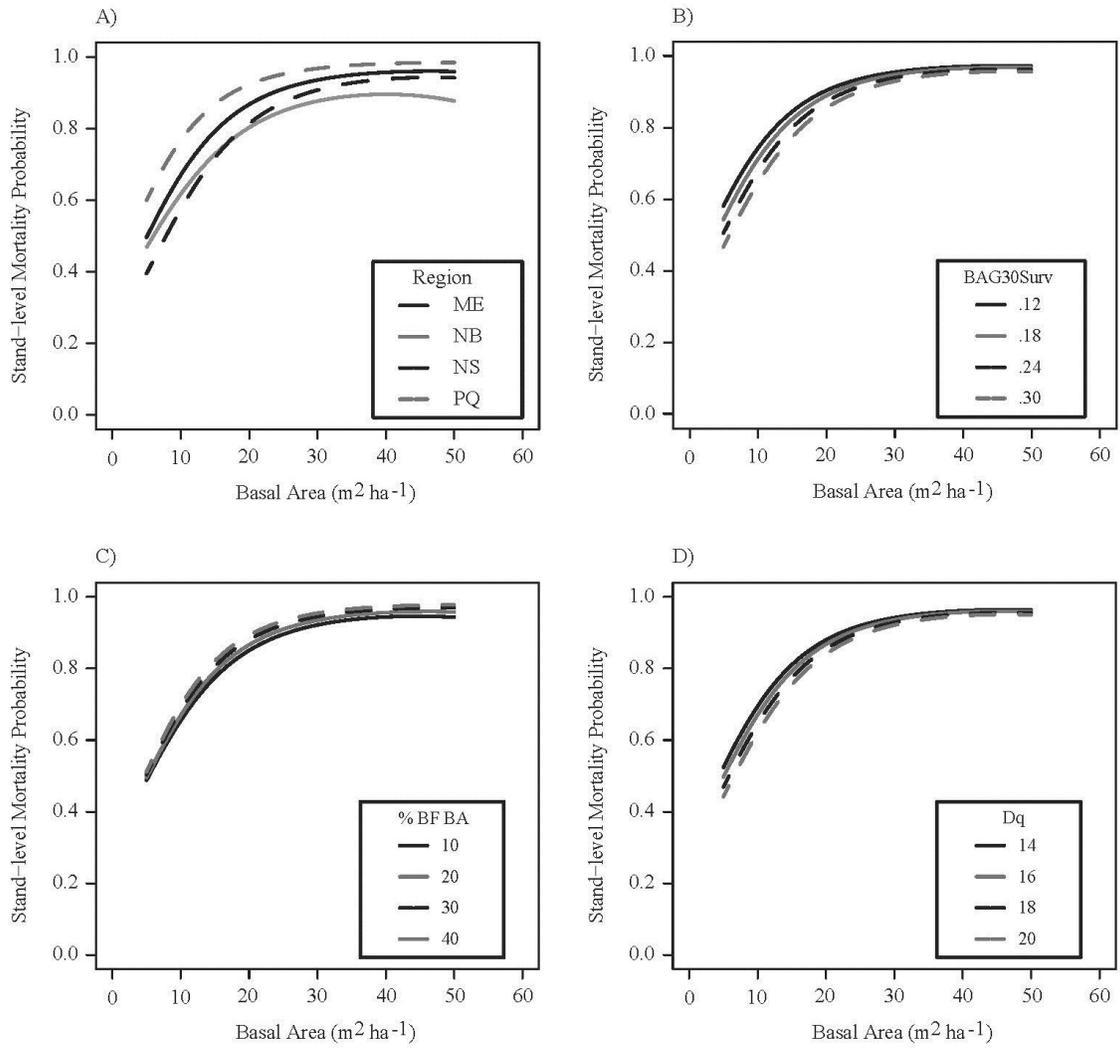


Figure 3. Predictions of stand-level basal area mortality given its occurrence using Eqn. 7 for the different regions (A), survivor basal area growth rate (B), percent balsam fir basal area (C), and quadratic mean diameter (D) over stand basal area ($m^2 ha^{-1}$).

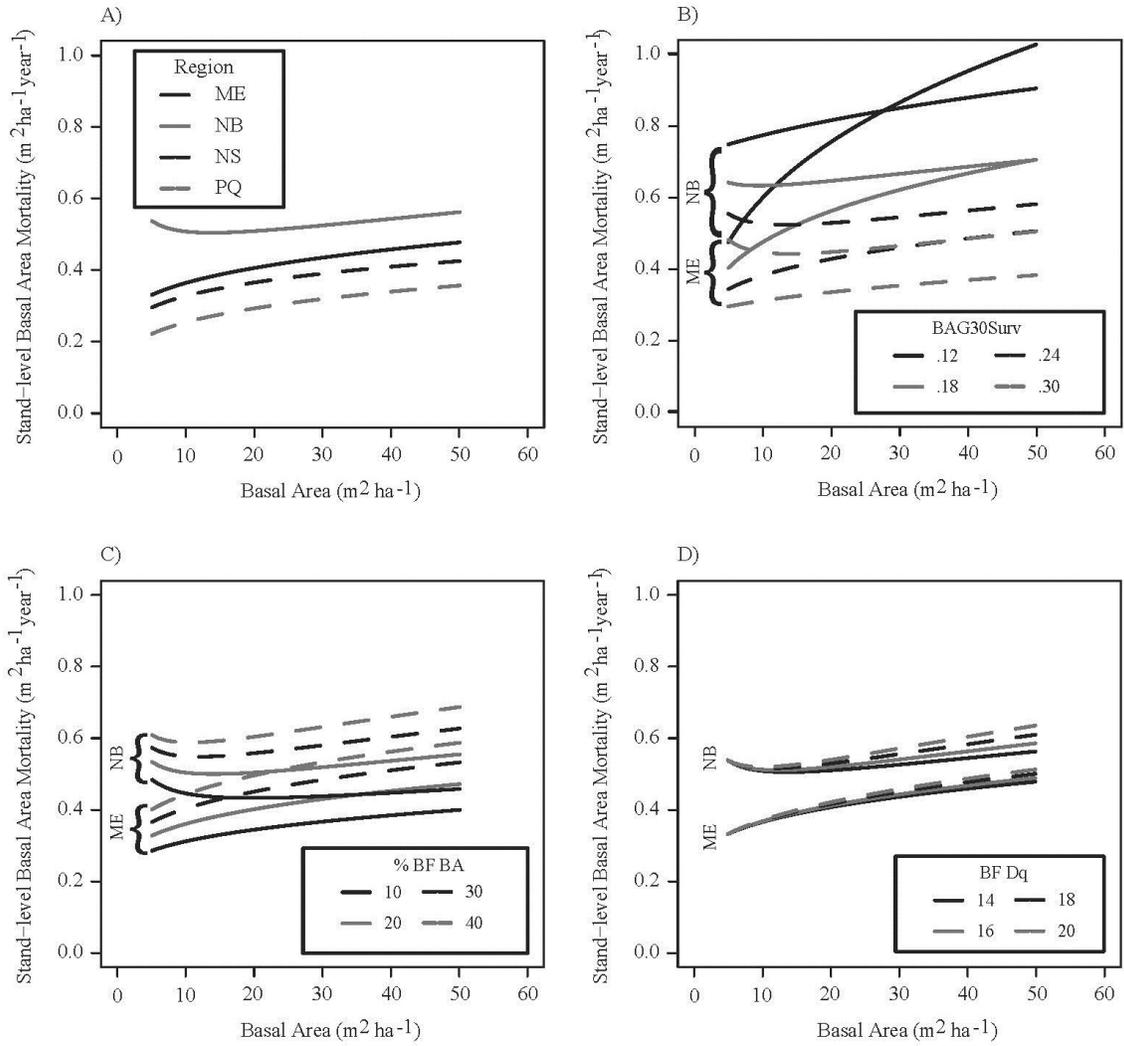


Figure 4. Predictions of stand-level basal area mortality given its occurrence using Eqn. 8 for the different regions (A), survivor basal area growth rate (B), percent balsam fir basal area (C), and balsam fir quadratic mean diameter (D) over stand basal area (m² ha⁻¹).

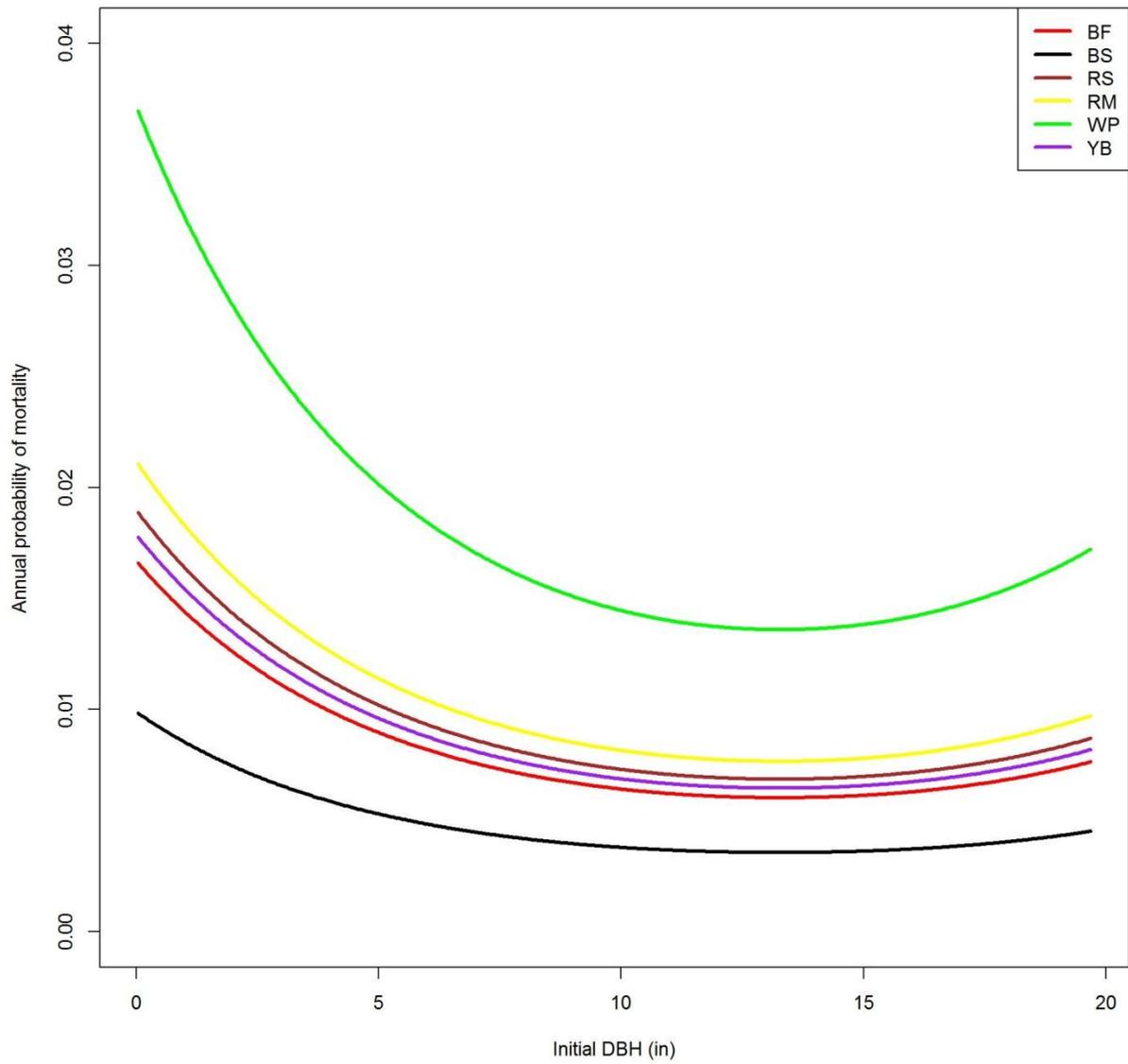


Figure 5. Annual probability of tree mortality over tree diameter at breast height (inches) using Eqn. 8 for selected species.

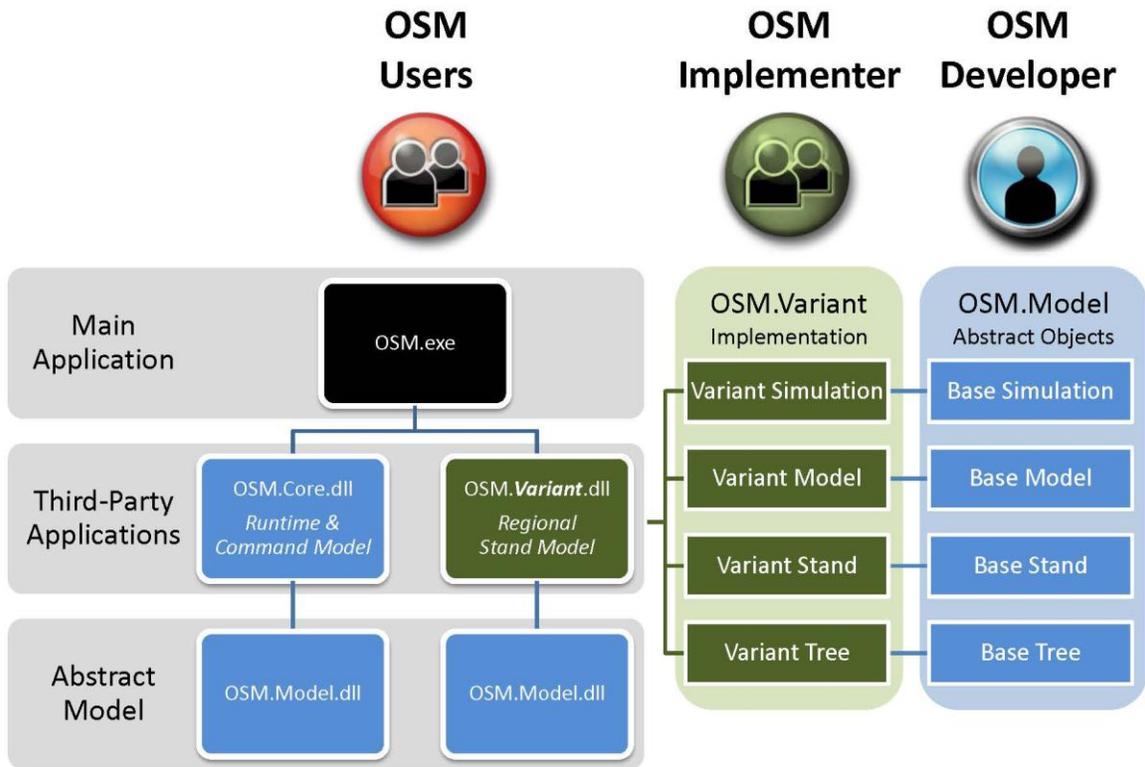


Figure 6. Structure and architecture of the Open Stand Model (OSM) currently being developed by Dr. Chris Hennigar.